A GARCH MODEL APPROACH TO CALCULATE THE VALUE AT RISK OF ALBANIAN LEK EXCHANGE RATE

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Abstract
The exchange rate regime applied in Albania is a flexible one, as a result it directly brings the exposure to exchange rate risk. Value at Risk (VaR) is one of the known measures of this type of risk. The measurement of VaR for the EUR/ALL exchange rate is the aim of this paper. Taking into the consideration that Albania has its trade activity mainly with countries that are part of Eurozone and that Euro is actually the main reserve value tool, we are focused only in this currency. The GARCH model is used as the approach to measure VaR, as in our opinion, it is the most appropriate for this problem. The results confirm the stability of the model and it being appropriate. This paper is an original attempt to use this method for the calculation of VaR for the EUR/ALL exchange rate.

Keywords: Exchange rate risk, value at risk (VaR), GARCH

Problem statement
Exchange rates are one of the main issues of debates between economists when it comes to discuss the problems concerning foreign trade of a country. The accumulated economic theory regarding international trade considers the exchange rate as one of the factors with the greatest importance and effect on defining the orientation and volume of trade with other countries (Dornbusch and Fischer 2000).

Albania has adopted a flexible exchange rate regime since the beginnings of the economic transition, when it entered into a free market economic system (Vika and Luçi 2011). The exchange rate has not reflected the proposed relation from current theory; said differently the trade balance is not significantly affected by the exchange rate. This comes from the fact
that, maybe, competition of our economy is not yet integral with regional trade or even wider with international trade (Salko et al. 2012). Proofing this hypothesis may be very interesting but we are not focusing on this aspect. On the other side, it can be thought that the decrease of the effect of the exchange rate may have come from the policies followed by the Bank of Albania, which intervenes in the market in order to orient the local currency markets (Çera, et al. 2012).

The exchange rate is one of the macroeconomic indicators that itself carries uncertainty typical of a financial indicator (Hoda 2012). The volatility of the exchange rate is inevitably related with the risk exposure of changing exchange rates. The exposure towards changing exchange rates of foreign currencies is the measure in which monetary flows, coming from transactions in foreign currency, are sensible to the changing of exchange rates of that currency. Economical units, based on demand and supply for foreign currency, are divided in four categories: exporters, importers, foreign investors and speculators (Salko, Dhuci & Kola 2010). In the following table we have presented the variance-covariance matrix for the bid exchange rate of various currencies against the Euro, and in the last row we have integrated the variance of the bid-ask spread, which is calculated as:

\[
\text{spread} = \frac{\text{ask} - \text{bid}}{\text{ask}} \times 100
\]

Table 1. The variance-covariance matrix for bid exchange rate and variance of spread for each currency. Data time horizon is 1.1.2002-2.12.2012. Authors’ calculations. Source: OANDA.

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>ALL</th>
<th>GBP</th>
<th>CHF</th>
<th>JPY</th>
<th>CNY</th>
<th>MKD</th>
<th>TRY</th>
<th>HRK</th>
<th>RSD</th>
<th>HUF</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL</td>
<td>0.027</td>
<td>59.523</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GBP</td>
<td>0.009</td>
<td>0.403</td>
<td>0.008</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHF</td>
<td>0.000</td>
<td>-0.840</td>
<td>-0.007</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPY</td>
<td>0.940</td>
<td>-104.30</td>
<td>-0.629</td>
<td>2.235</td>
<td>334.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CNY</td>
<td>0.084</td>
<td>-3.918</td>
<td>-0.012</td>
<td>0.089</td>
<td>14.242</td>
<td>0.905</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKD</td>
<td>0.019</td>
<td>3.001</td>
<td>0.034</td>
<td>-0.042</td>
<td>-5.625</td>
<td>-0.175</td>
<td>0.822</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRY</td>
<td>0.028</td>
<td>1.249</td>
<td>0.022</td>
<td>-0.026</td>
<td>-2.029</td>
<td>-0.023</td>
<td>0.064</td>
<td>0.086</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HRK</td>
<td>0.006</td>
<td>0.271</td>
<td>0.004</td>
<td>-0.007</td>
<td>-0.542</td>
<td>-0.007</td>
<td>0.007</td>
<td>0.019</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RSD</td>
<td>1.272</td>
<td>57.95</td>
<td>1.119</td>
<td>-1.459</td>
<td>-133.9</td>
<td>-2.851</td>
<td>4.079</td>
<td>3.772</td>
<td>1.128</td>
<td>242.81</td>
<td></td>
</tr>
<tr>
<td>HUF</td>
<td>0.752</td>
<td>69.88</td>
<td>1.106</td>
<td>-1.533</td>
<td>-178.1</td>
<td>-5.963</td>
<td>3.240</td>
<td>3.906</td>
<td>1.157</td>
<td>210.65</td>
<td>304.26</td>
</tr>
</tbody>
</table>

Variance of spread: \( \text{spread} = \frac{\text{ask} - \text{bid}}{\text{ask}} \)

| Var.  | 0.0002 | 0.0246 | 0.0005 | 0.0003 | 0.0003 | 0.0012 | 0.0222 | 0.0137 | 0.0322 | 0.0212 | 0.0034 |

The diagonal of the matrix shows the variance of the bid exchange rate for a given currency, or stated differently the volatility of the respective currency, while the numbers below the diagonal represent the covariance between the two respective currencies, which reports the direction of

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42 We noticed that the same considerations can be said also for the ask exchange rate.
relationship between the currencies. As we can see, the ALL (Albanian Lek) has a negative relationship with the JPY and the CNY and positive relationship with the greatest part of the foreign currencies taken into consideration. This relation can be supported by the gravity model, which stresses that the trade volume can be evaluated by an increase of the size of the economy or national income of partner countries, and as a decreasing function of the distance between them (Skreli and McCalla 2012). We can notice that the Albanian Lek has a strong volatility (only the Japanese Yen, the Hungarian Forint and the Serbian Dinar have a higher volatility). Of a great importance is the fact that the variance of the spread of the EUR/ALL exchange rate is the highest, 0.0246 (except for the EUR/HRK rate). Both the direction of the relationship and the variance of the spread are confirmed by theory in this field.43

As we saw above, it results that the Albanian Lek is one of the most volatility currencies, which can bring to a high variance of the spread, affecting in this way the interests of the economic units that deal with currency, like exchange offices or agencies. According to the publications of the Bank of Albania, the number of exchange offices in Albania is 323. On the other side, the number of banks that operate in Albania is actually 16, and the total loan portfolio (December 2012) was the equivalent of 540,041.38 million ALL, from which 62.06% is in foreign currency, where the major part is in Euro, 55.26%. This high proportion of foreign currency loans demonstrates the high level of exposure of the banking system to the exchange rate risk. The same thing can be said for the foreign trade sector, which accounts for a considerable part of the local GDP (exports about 15% and imports about 40%), exposing the economy also to the currency risk.

Questions that arise are, is it possible to measure the exposure to the exchange rate risk? Can we measure the maximum value that is expected to be lost during unfavorable volatilities of exchange rates? In other words, it is requested to determine the value at risk (VaR) of the volatilities of the exchange rate. As there are many methods for calculating VaR, our goal is to use a methodology for the measurement of VaR, in order that it becomes not only a good measure of risk, but results to be stable in time, and not affected by the period of time taken into consideration.

**Methodological aspects**

The measurement of VaR of the exchange rate EUR/ALL through GARCH modeling is the goal of this paper. The most spread definition of VaR is: *value at risk is the greatest monetary amount that can be lost during*

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43 The spread of an exchange rate depends on the level of marketability and volatilities of the respective rate and its trading commissions (Salko, Dhuci and Kola 2010, 91).
a given time period with a given confidence level (Jorion 2001). So VaR is both a financial concept with strong risk doses and also a statistical one.

Let us denote the exchange rate EUR/ALL with \( k \), while its return with \( r \), and we can use the natural logarithm of the ratio of the daily exchange rate with that of a day before to calculate the return:

\[
\begin{align*}
  r_t &= \ln\left(\frac{k_t}{k_{t-1}}\right) = \ln k_t - \ln k_{t-1} \\

\end{align*}
\]

**Data** for the exchange rate of ALL with Euro are daily averages and are presented in the form of dynamic series distributed over the time horizon January 1, 2002 up to June 20, 2013.\(^{44}\) They are taken from the official database of the Bank of Albania. The total number of records for exchange rates is 2865.

The method that we have used to calculate the value at risk of the exchange rate between ALL and EUR is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modeling, (Bollerslev 1986, Engle 2001). This model is part of the ARCH family models (Autoregressive Conditional Heteroskedasticity), where this last one derives from an AR (Auto Regressive) modeling of a dynamic series. Let the AR process be,

\[
Y_t = \alpha_0 + \sum_{i=1}^{d} \alpha_i Y_{t-i} + \varepsilon_t
\]

From this process is taken and modeled the variance of the error term through the remaining term, so the ARCH model is evaluated in this way:

\[
\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i}
\]

When an ARMA model (Auto Regressive Moving Average) has errors in variance, then the model is GARCH (Bollerslev 1986). As the forecasting of variance is based on information of the past, then this is known as conditional variance. In these cases, the model which is GARCH(p, q), where \( p \) is the GARCH order of \( \sigma^2 \) terms and \( q \) is the ARCH order of the error terms, can be written as:

\[
\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i}
\]

The evaluation of the parameters of this type of model is considered the most difficult and delicate moment. The difficulty becomes greater when we learn that the variance (\( \sigma^2 \)) is unobservable, is a latent variable. In these cases the likelihood method is used. If we are based at the normal

\(^{44}\) The Bank of Albania archives daily average exchange rates of ALL with other currencies 4 or 5 days for each week.
distribution, then the coefficients of the GARCH(1,1) model are estimated through the formula (Bollerslev 1986):

\[
\max F(\omega, \alpha, \beta | \epsilon) = \sum_{i=1}^{n} \ln f(\epsilon_i | \sigma_i^2) = \sum_{i=1}^{n} \left( \ln \frac{1}{\sqrt{2\pi \sigma_i}} - \frac{\epsilon_i^2}{2\sigma_i} \right)
\]

subject to \[
\begin{align*}
  p & \geq 0, q > 0, \\
  \omega & > 0, \alpha_i & \geq 0, i = 1, \ldots, q \\
  \beta_i & \geq 0, i = 1, \ldots, p
\end{align*}
\]

It is proofed a relationship between the values of the parameters and unconditional variance (Bollerslev 1986). For GARCH(1,1), it is given by the relation:

\[
\sigma^2 = \frac{\omega}{1 - \alpha - \beta}.
\]

Value at risk has been discussed also by many other authors either for exchange rates or for security portfolios, and there exists a well defined relationship between VaR and the exchange rate. This increases the clarity level of our problem.

Keeping in mind that this topic interests both to academics and business subjects, we judge that this paper aimed at measuring VaR is addressed to both. Through this paper we have attempted to add something on this topic in the case of Albania, bringing to the attention of the reader that GARCH modeling can be a very valuable and successful tool for determining the VaR of the exchange rate.

**Literature review**

We would like to bring to the attention of the reader the contribution of other authors, which have dealt with a topic similar to this, and to position our research on it. It is important to stress that VaR can be determined in several ways. Theoretically it is widely known that VaR can be determined through the following methods:

1) The historical method, which is based on historical data with a determined time horizon and with the assumption that past events can be repeated in the future;
2) The variance-covariance method, which is based on the assumption that the variable that is studied has a normal distribution;
3) The simulating method, which uses a model to generate random hypothetical values.

GARCH modeling uses historical data to evaluate the conditional variance and through it to evaluate VaR. The most cited author about the use of the GARCH modeling for calculation of VaR is Philippe Jorion, and his
book “Value at risk: the new benchmark for managing financial risk” is considered as a “guru” when it comes to this topic. In year 1986 the GARCH modeling approach was formulated by Tim Bollerslev, which was preceded by the ARCH approach presented by Robert Engle in year 1982.

Recently, the GARCH modeling approach is one of the most used econometric tools, related to the measurement of volatilities of financial indicators (Engle 2001, Jorion 2003, Žiković 2008, Francq and Zakoian 2010). This method has often been used by many authors either for calculation of exchange rate VaR, or for the VaR of a stock or a security portfolio. During our research we have not found any paper or discussion material with a focus on the VaR of the exchange rates in the case of Albania. This makes this research original by using GARCH in calculating the VaR of the EUR/ALL exchange rate.

GARCH modeling to calculate the VaR of the return of the exchange rate

In the literature it is often mentioned the GARCH(1,1) modeling as the most appropriate model to measure the volatilities of the return of the exchange rate (Engle 2001, Dowd 2002, Resti and Sironi 2007, Žiković 2008, Erdemlioglu, Laurent and Neely 2012, Jorion 2001) and its mathematical form is written as:

\[ h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1}. \]

Before the estimation of the parameters of the model \( \omega, \alpha \) and \( \beta \), let us stop a bit at the distribution of the \( r \) series. The graph below gives the illustration of the performance of the exchange rate EUR/ALL, which we have denoted as \( k \) series, and of the series of return of exchange rate that we have denoted with \( r \) in the time period starting from January 1, 2002 up to June 20, 2013.

![Figure 1](image_url)

**Figure 1.** Graphic illustration of the performance of the \( k \) (LHS) and \( r \) series (RHS).

45 Where \( h \) represents the variance, \( \sigma^2 \).
The ADF test shows that the $r$ series is stable, because the ADF test statistic is greater than the 1%, 5% or 10% critical value. The table below shows it in details.

**Table 2. The unit root test of $r$ series.**

<table>
<thead>
<tr>
<th>ADF test statistic</th>
<th>1% Critical value</th>
<th>5% Critical value</th>
<th>10% Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-25.98401</td>
<td>-3.4357</td>
<td>-2.8631</td>
<td>-2.5676</td>
</tr>
</tbody>
</table>

The distribution of $r$ series is wider than the normal distribution, because the kurtosis coefficient is greater than 3, and is skewed as the skewness coefficient is not 0.\(^{47}\) The average of the $r$ series is almost 0. The table below shows more information about this discussion.

**Table 3. Some statistical data for the $r$ series.**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Value</th>
<th>Indicator</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.81E-5</td>
<td>Standard deviation</td>
<td>0.002921</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.020497</td>
<td>Skewness</td>
<td>0.264415</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.019296</td>
<td>Kurtosis</td>
<td>9.319288</td>
</tr>
</tbody>
</table>

The $r$ series suffers from the ARCH effect, because the autocorrelation coefficients are statistically significant (all the $p$-values are 0 with three decimal digits, except for the first order which is 0.002). The first order of autocorrelation is 0.059 and gradually decreases to -0.016 in the 16-th order.\(^{48}\) This is also shown by the following table. We expected the ARCH effect, because even the amplitude of the return varies over time (Engle 2001).

**Table 4. Autocorrelation coefficients and their testing for the first 16 orders.**

<table>
<thead>
<tr>
<th>Order</th>
<th>Autocorrelation</th>
<th>Partial correlation</th>
<th>Q-Stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.059</td>
<td>0.059</td>
<td>10.016</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>-0.066</td>
<td>-0.070</td>
<td>22.453</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>-0.070</td>
<td>-0.063</td>
<td>36.683</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>-0.025</td>
<td>-0.021</td>
<td>38.407</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>-0.009</td>
<td>-0.016</td>
<td>38.651</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.035</td>
<td>0.030</td>
<td>42.245</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.004</td>
<td>-0.005</td>
<td>42.281</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>-0.012</td>
<td>-0.010</td>
<td>42.712</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.018</td>
<td>0.023</td>
<td>43.669</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>-0.012</td>
<td>-0.015</td>
<td>44.116</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>0.006</td>
<td>0.010</td>
<td>44.215</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>-0.044</td>
<td>-0.046</td>
<td>49.814</td>
<td>0.000</td>
</tr>
<tr>
<td>13</td>
<td>-0.003</td>
<td>0.002</td>
<td>49.845</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\(^{46}\) All the following tables are generated using EViews.

\(^{47}\) As this coefficient is positive, we reach to the conclusion that the right side of the distribution is more extreme, the distribution is skewed on the right.

\(^{48}\) In these cases it is referred about the decrease of the autocorrelation coefficient in absolute value.
In the summary table of the GARCH(1,1) model, are given some model and parameters diagnostic tests. Based on it we build the model for $h$:

$$h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1} \Rightarrow h_t = 2.14 \times 10^{-6} + 0.15 r_{t-1}^2 + 0.599999 h_{t-1}$$

### Table 5. Summary of model GARCH(1,1).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C = $\omega$</td>
<td>2.14E-06</td>
<td>1.09E-06</td>
<td>1.958014</td>
<td>0.0502</td>
</tr>
<tr>
<td>ARCH(1) = $\alpha$</td>
<td>0.150000</td>
<td>0.029542</td>
<td>5.077537</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(1) = $\beta$</td>
<td>0.599999</td>
<td>0.064030</td>
<td>9.370630</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

- **$R^2$** = -0.00027
- **AIC Criterion** = -9.11213

**Log likelihood** = 13051.57

Durbin-Watson statistic = 1.881034

**Note:** Dependent variable: $r$
Method: ML - ARCH
Sample: 2864 after adjustments
Convergence achieved after 1 iterations.

Coefficients $\alpha$ and $\beta$ are statistically important even at 99% confidence level. The intercept ($\omega$) becomes important at 94.98% confidence level. The condition that the sum $\alpha + \beta$ should be less than 1 is met, because $(0.15 + 0.599999) < 1$, and as a result we can say that the series obtained from the model GARCH(1,1) is a stationary process (Jorion 2001). Also, the unconditional variance results to be $8.55997E-06$, which Engle (2001) identifies as the average variance of the process in the long term.\(^{49}\)

Let us build the graph of the VaR for the return of the exchange rate by using the model GARCH(1,1). The estimated model above helps us to determine the conditional variance ($h$) of the $r$ series. We need to determine the VaR of this return. We will do this by using the VaR formula related with $h$ and a level of confidence of ($\gamma = 1 - \alpha$), which is:

$$VaR_\alpha = CP \times Z_\alpha \sqrt{h_t}$$

where $CP$ is the monetary amount of the currency position, $Z_\alpha$ is the value of the normal distribution $N(0,1)$ for the level of probability $\alpha$ and $h_t$ is the conditional variance that has been estimated through the GARCH(1,1) model (Jorion 2003). If we need to calculate the VaR for 95% confidence level for the return of the exchange rate of a currency position of 1 EUR, then the formula would be:

$$VaR_\alpha = PV \times Z_\alpha \sqrt{h_t} \bigg|_{PV = 1, \alpha = 0.05} = 1 \times 1.65 \sqrt{h_t} = 1.65 \sqrt{h_t}$$

\(^{49}\) Remember that the formula of unconditional variance is, $h = \omega/(1 - \alpha - \beta)$. 

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The graph below illustrates the relation of the VaR with the return of exchange rate $r$. The line that follows the relationship $VaR_\alpha = -1.65\sqrt{h_t}$ is exactly the maximal monetary amount that can be lost with a level of confidence of 95% over the period of time taken into consideration, when the business events are not favorable for the business activity. We bring into attention the fact that the maximal loss for the year 2012 results to be moderated. This may happen because our indicator for the year 2012 does not reflect the volatilities of previous periods.

The importance of this graph or of the VaR relation with $r$ stays in the fact that exchange offices, banks and businesses that operate in Albania and have transactions in Euro, using this formula can easily calculate the VaR of every transaction. Besides this, those subjects that actively manage the exchange rate risk, can use the relationship to limit their positions, or to build and operate a policy of risk management for the exchange rate risk, in order to limit their losses, as oriented by their overall strategy. Risk budgeting through the VaR requires a precision and a good level of approximation of the VaR so that it can be efficient.

The calculation of VaR through the use of the GARCH model has also advantages of the nature of forecasting the values of the value at risk in the future. If the other factors remain constant, then the GARCH model gives a very high level of approximation with the real values of the VaR.

**Conclusions and discussions for further research:**

Value at risk is one of the techniques that presents much interest to be evaluated and mostly used when we speak for the effects that unfavorable risk events can have on a currency position. Recent developments in international economy strengthen more the need to exactly calculate VaR. GARCH modeling helps us to measure VaR and, referred to our research, we can consider it as the first effort to determine the VaR of the return of the exchange rate EUR/ALL. The advantage of the application of such a model stays in the fact that it gives results based on historical data, results that are
tested statistically, and the possibility for forecasting the performance of VaR in the future, with the condition that the other factors remain constant.

Based on what we said above, we conclude that the GARCH modeling approach for the calculation of VaR can serve to individuals and businesses that are exposed to volatilities of the EUR/ALL exchange rate. We consider this as the most important finding of this paper.

As a discussion for further research, we think that it still remains to be seen which is the most appropriate model to measure the VaR of the exchange rate of ALL with other currencies besides EUR, and if the currency risk exposure can be decreased by diversifying the portfolio in various currencies.

References:


OANDA. *Currency converter*. n.d. 


